

[trig identities]

1. Prove the following identities:

a. $\sin \theta \tan \theta + \sec \theta = \frac{\sin^2 \theta + 1}{\cos \theta}$

b. $\sin^2 \theta \cot^2 \theta = 1 - \sin^2 \theta$

c. $\csc^2 \theta - 1 = \csc^2 \theta \cos^2 \theta$

d. $\sin^2 \theta = \frac{\tan^2 \theta}{1 + \tan^2 \theta}$

e. $\frac{\sin \theta + \cos \theta \cot \theta}{\cot \theta} = \sec \theta$

f. $1 - \sin \beta \cos \beta \tan \beta = \cos^2 \beta$

g. $\frac{\sin \theta + \tan \theta}{1 + \cos \theta} = \tan \theta$

h. $(1 - \cos \theta)(1 + \cos \theta) = \frac{1}{\csc^2 \theta}$

i. $\cos^3 \theta + \cos \theta \sin^2 \theta = \frac{1}{\sec \theta}$

j. $\frac{\cos \theta}{1 + \sin \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$

k. $\tan \theta + \cot \theta = \sec \theta \csc \theta$

l. $\cos^4 \theta - \sin^4 \theta + 2 \sin^2 \theta = 1$

m. $\cos^2 \theta = (1 - \sin \theta)(1 + \sin \theta)$

n. $\frac{\sin^2 \theta}{1 - \cos \theta} = 1 + \cos \theta$

o. $\frac{1 - \tan^2 \beta}{\tan \beta - \tan^2 \beta} = 1 + \frac{1}{\tan \beta}$

p. $1 - \cos^2 \theta = \frac{\cos \theta \sin \theta}{\cot \theta}$

q. $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$

r. $\frac{1}{1 + \tan^2 x} + \frac{1}{1 + \cot^2 x} = 1$

s. $\frac{\sec \theta}{\csc^2 \theta} = \sec \theta - \cos \theta$

2. Solve the following for the angle, on the interval $[0^\circ, 360^\circ]$

a. $2 \cos \alpha = 1$

b. $5 \sin \theta = 4$

c. $2 \cos^2 \beta = 1$

d. $\sin^2 x + \sin x = 0$

e. $2 \cos^2 \theta - \cos \theta = 0$

f. $\sin 2\alpha = 0.75$

g. $4 \cos 2\theta - 3 = 0$

h. $2 \cos^2 \beta + \cos \beta - 1 = 0$

i. $3 \tan \theta - 2 = 0$

j. $5 \tan 2x + 2 = 0$

k. $2 \sin x = \cos x$

l. $\csc \theta = 1$