[exponential + logarithm functions]

1. Evaluate:
   a. -3^2
   b. $(-3)^{-2}$
   c. $\left(\frac{1}{5}\right)^{-2}$
   d. $27^{\frac{2}{3}}$
   e. $128^{-\frac{3}{7}}$
   f. $\left(\sqrt[5]{32}\right)^2$
   g. $\frac{24^3}{12^5}$
   h. $\frac{216^2}{36^3}$
   i. $\frac{8^{-4}}{16^{-3}}$
   j. $\left(\sqrt[4]{81}\right)^{-2}$
2. Evaluate:
   a. $\sqrt{\frac{4}{49}}$
   b. $\left(\frac{4}{7}\right)^{-1}$
   c. $\sqrt{\left(\frac{4}{25}\right)^{-1}}$
   d. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$
   e. $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$
3. Evaluate:
   a. $\frac{8^{\frac{1}{3}}+81^{\frac{1}{4}}}{121^{\frac{1}{2}}-3^{\ 0}}$
   b. $\frac{2^{-1}+3^{-1}}{4^{-1}}$
   c. $\frac{4^0}{2^{-3}-3^{-2}}$
4. Evaluate:
   a. $\frac{2^{-3}\cdot 3^{-2}\cdot 4^{-\frac{3}{2}}}{8^0\cdot 9^{-1}\cdot 8^{-\frac{5}{3}}}$
   b. $8^{\frac{2}{3}} - 11^0 + \left(\frac{4}{9}\right)^{-\frac{3}{2}}$
5. Simplify and express the answers with **positive** exponent
   a. $3a^{-1}b^{-2}c^3$
   b. $\frac{a^{-2}b^3c^{-1}}{a^{-3}bc^2}$
   c. $(2a^{-1}b^2)^{-3}$
   d. $\left(\frac{2a^{-1}}{3b}\right)^{-2}$
   e. $(a^{-2}+b)(a^{-3}-b^{-2})$
   f. $(a^{-3}-a^3)^2$
   g. $(8a^6b^{-9})^{\frac{2}{3}}$
   h. $(25a^{-2}b^4)^{-\frac{1}{2}}$
6. Simplify fully:
   a. $x^{a-b}\cdot x^{a+b}$
   b. $\frac{x^{a-b}}{x^{a+b}}$
   c. $9^x 27^x$
   d. $\left(\frac{x}{y}\right)^a\left(\frac{y}{x}\right)^{2a}(xy)^b$
   e. $\frac{(x^3)^a(x^b)^2}{(x^3)^c}$
   f. $(x^a)^b \div x^c$
   g. $\frac{8^x\cdot 16^y}{32^z}$
   h. $\frac{9^a\cdot 3^{a+2}}{27^{a-1}}$
   i. $3^a + 3^a + 3^a$
   j. $\frac{1+a^{-1}}{1-a^{-1}}$
   k. $\frac{x^{-1}-y^{-1}}{x^{-1}-y^{-1}}$
   l. $\frac{x^{-1}+y^{-1}}{(x+y)^{-1}}$

7. Given $2 = 10^{0.301}$ and $3 = 10^{0.477}$ and $5 = 10^{0.699}$ express each of the following as a power of 10:
   a. 6
   b. $\frac{2}{3}$
   c. 8
   d. 7.5
   e. $\sqrt[3]{5}$
8. The number of bacteria in a culture doubles every 2 hours. Find the number of bacteria present after 3 hours and after n hours, if there were originally 100 bacteria.
9. In an experiment, a culture of 1000 bacteria grew to 16 000 in 2 hours. Find the doubling rate and the number of bacteria after n hours.
10. If principal P earns interest i, compounded n times, the final amount A of the investment is $A = P(1 + i)^n$. Find A if $P = \$1000$, $i = 8\%$ per year, and $n = 10$ years.
11. Write the statements in logarithmic form:
   a. $2^3 = 8$
   b. $3^{-2} = \frac{1}{9}$
   c. $16^{\frac{1}{4}} = 2$
12. Write these statements in exponential form:
   a. $\log_5 125 = 3$
   b. $\log_4\left(\frac{1}{64}\right) = -3$
   c. $\log_{49}\left(\frac{1}{7}\right) = -\frac{1}{2}$
13. Find these logs:
   a. $\log_2\left(\frac{1}{8}\right)$
   b. $\log_3(9\sqrt{3})$
   c. $\log_5\left(\frac{\sqrt{5}}{125}\right)$
   d. $\log_3 9^4$
   e. $\log_{\frac{1}{2}} 2$
   f. $\log_{\frac{1}{3}} 27$
   g. $\log_{\frac{2}{3}}\left(\frac{8}{27}\right)$
   h. $\log_{\frac{2}{3}}\left(\frac{9}{4}\right)$
   i. $\log_{\frac{4}{25}}\left(\frac{5}{2}\right)$
14. Given that $\log_{10} 2 = 0.301$ and $\log_{10} 5 = 0.699$. Find:
   a. $\log_{10} 10$
   b. $\log_{10}\left(\frac{2}{5}\right)$
   c. $\log_{10} 2.5$
   d. $\log_{10} 10000$
   e. $\log_{10} 125$
   f. $\log_{10}\left(\frac{1}{8}\right)$
   g. $\log_{10} 0.001$
15. Evaluate:
   a. $7^{\log_7 3}$
   b. $5^{\log_5 2 + \log_5 3}$
   c. $3^{4\log_3 2}$
   d. $6^{\log_6 8 - \log_6 2}$
16. Solve for x:
   a. $8^x = 4^{x-1}$
   b. $\left(\frac{1}{9}\right)^{x+2} = \left(\frac{1}{27}\right)^x$
   c. $5^{x+2} = 1$
   d. $25^{x+2} = 625$
   e. $16^x \cdot 8^{x+1} = \frac{128}{32^{x-1}}$

17. Solve for x:
    a. $4\log_2 x = \log_2 16$
    b. $\log_3 \sqrt{x} = \log_3 5$
    c. $\log_x 16 = 4$
    d. $\log_x 25 = 4$
    e. $\log_x 16 = 3$
    f. $\log_2 3 = x$
    g. $\log_3 \left(\frac{1}{2}\right) = x$
    h. $\log_{\frac{3}{5}} 4 = x$

18. Solve for x:
    a. $\log_3 x + \log_3 5 = \log_3 20$
    b. $\log_2 x - \log_2 3 = 3$
    c. $\log_4(x-3) - \log_4 x = 2$
    d. $\log_3 x + \log_3(x-2) = 1$
    e. $\log_5(x+2) + \log_5(x-3) = \log_5 6$
    f. $2\log_6(x-3) = \log_6(5-x)$